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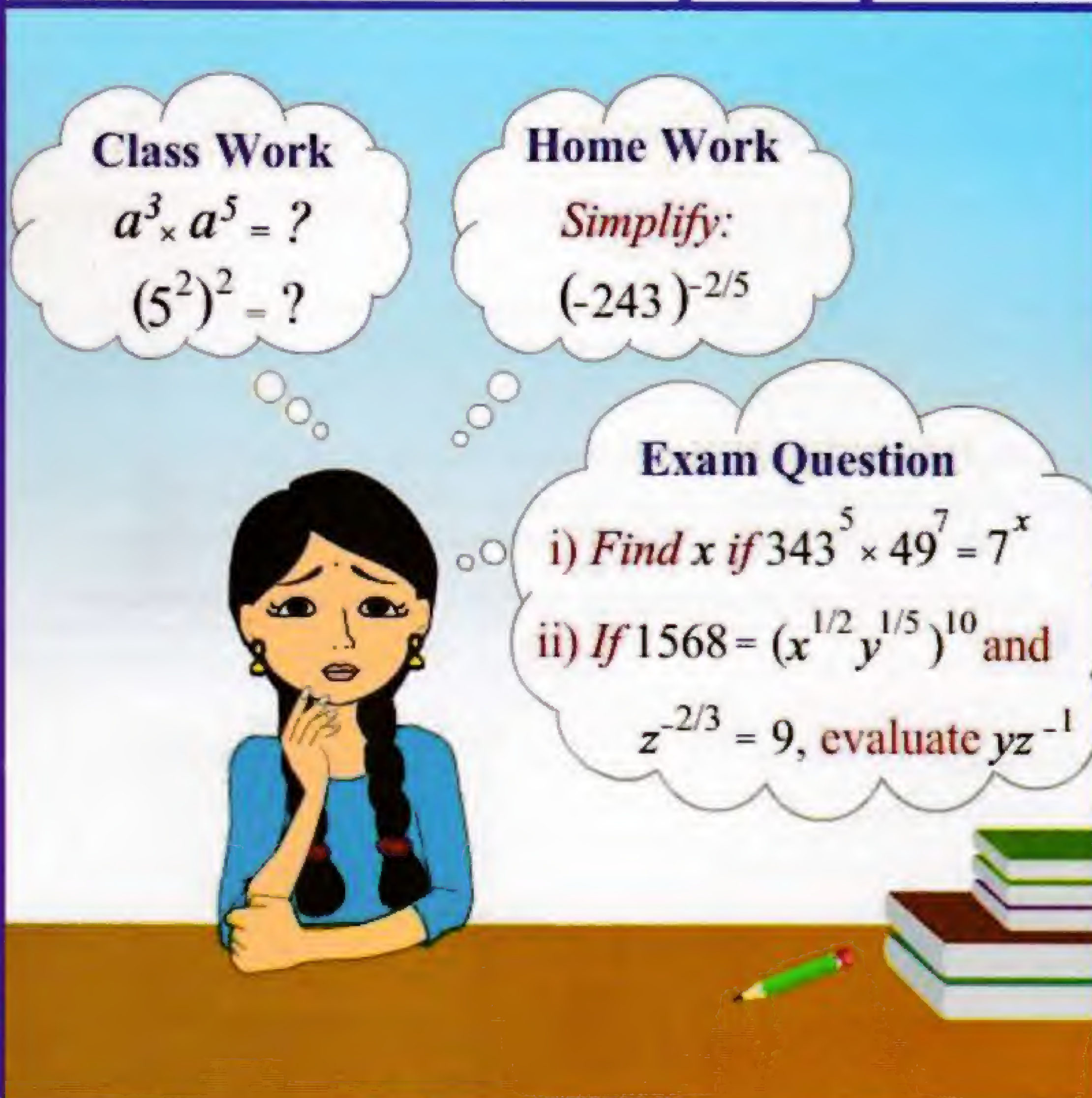
R. ATHMARAMAN



JM

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Junior mathematician (JM) is a Mathematics magazine, principally meant for youngsters of age between 8 and 16.

- Aims to interact directly with the fresh, young and receptive minds, motivating them in their appreciation and application of Mathematics.
- Aspires to present Mathematics as a lovable subject, satisfying to the serious-minded and pleasurable for others, removing math-phobia.
- Provokes young student-authors to write their own discoveries and creative thoughts.

Junior Mathematician comes out essentially as a journal for retreat, in a volume of three issues annually as follows:

No.1 in September, No.2 in December and No.3 in March (a double issue to celebrate summer holidays of the school-going)

Articles for the magazine and all correspondence regarding publication material need to be sent to "the Editor: Junior Mathematician" at the address of the AMTI. The Editor is also available on e-mail at athmaramanr@gmail.com and phone 09444611066.

All communications regarding subscriptions, receipt of issues, change of address and other administrative matters should be addressed to "The General Secretary: AMTI, B-19, Vijay Avenue, 85/37, Venkatrangam Pillai Street, Triplicane, Chennai-600005.

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From the Editor's Desk:

A REMINDER TO CONTRIBUTORS

A good account of the nature of articles which we entertain at the editorial section has been already brought to the attention of our readers. We need simple and purposeful pieces of mathematical encounters. Still we receive articles that are quite artificial, with matter drawn liberally from the websites, not even sparing diagrams, colour or titles; they are of no use to the readers. While referring to the web resources has become unavoidable, it is expected that an article submitted must reflect the author's own experience, investigation, reason for appreciation of certain concepts or methods etc; the aim is to share views on skills and concepts that would help better comprehension and admiration of mathematics. Secondly, JM is not for talking about very high level mathematics. This forum is for junior level students.

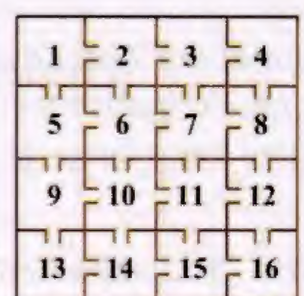
Often authors inquire why their articles have not been accepted for publication. We regret to inform that it is not possible to reply individually each such query.

QUERIES FROM READERS

(In this section we try to comment on some of the queries raised by the readers, choosing a few of them. Some serious readers have started to seek the help of JM to comprehend the concepts in which they have doubts; this is a heartening response to our efforts. It is not possible to discuss all the questions received; only a few selected among them could find a place in this section, as and when published. For these people, e-mail has been an ideal mode of communication.)

1. *R. Shivshankar from Tuticorin has the following question*:

I came across a problem that was suggested by my classmate. I tried to solve it by guessing, trial and error method. It runs as follows:



A hotel has 16 rooms, all interconnected, as shown in the figure. The manager of the hotel wants to travel through all the rooms to supervise the work done by his servants, not revisiting the room he has already seen. He wants to see room number 16 as the last. How can he do it?

After a lot of working with several alternatives, I got this way out: 13→9→5→1→2→6→10→14→15→11→7→3→4→8→12→16.

My doubts are as follows:

i. I have not started with room number 1. Is it alright?
ii. Is there any other solution for the problem?

Ed: Your classmate must be a web-browsing craze, for, the above problem is not his original but seems to be a modification of an investigation from "Stella's stunners" (search on Google!).

Coming now to your doubts, the problem has not anywhere mentioned that the manager starts with room 1; it simply tells you that he has to cover all the 16 rooms. While attempting to solve problems, you should not think of unwarranted assumptions.

4→3→2→1→5→6→7→8→12→11→10→9→13→14→15→16 is one more solution (verify it!). Remember that there can be many solutions for a problem.

You may also like to extend the idea to a situation where the number of rooms could be 9 or 25 etc. Just try!

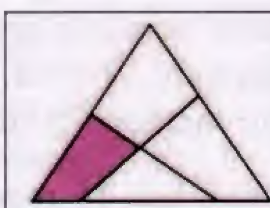
2. *K. Shobana from Chromepet writes*:

I have been told that every rational number can be expressed as a fraction. Are there fractions that are not rational numbers?

Ed: How about $\frac{\sqrt{3}}{2}$ or $\frac{\pi}{3}$?

3. *This one is by M. Balakrishanan from Gopalapuram*:

I am in my V standard class. This question was there in my Formative test. I answered 'Yes'. But my friend says I am wrong. Is he right or am I ?



Is the shaded part $\frac{1}{4}$?

Ed: You answered that the shaded portion is one-fourth of the whole triangle. Are you sure that the triangle has been divided into four *identical* parts with equal area?

4. *This is from Mrs Harini, a teacher from Valasaravakkam.* (A good question): **What is the opposite of Zero?**

The concept of opposite is a human invention. In nature, there is no real opposite. Is day opposite of night? Is male opposite of female? What is the opposite of color blue? Here we must be cautious when we ask about opposite of zero. One has to 'define' what one means by an opposite. The difference is between quality (a concept) versus quantity (which is a number). In the context of the real line, you can say that the opposite of zero is itself, while the opposite of +2 is -2 with respect to the origin point 0, etc., (one on its right-side and the other on the left-side). This definition is acceptable if you accept the opposite of left is the right. What is the opposite of ½? If you say, it is 2, then 0 has no opposite.

An unexpected solution!

Let x, y, and z be real numbers such that $x + y + z = 20$ and $x + 2y + 3z = 16$. What is the value of $x + 3y + 5z$?

Note that $(x+y+z)$, $(x+2y+3z)$, $(x+3y+5z)$ form an arithmetic sequence, giving us answer 12.

(You may also subtract as usual, the first equation from twice the second giving $x + 3y + 5z = 2(x + 2y + 3z) — (x + y + z) = 2(16) — 20 = 12$.)

A SUPER JUGGLERY

*S. A. RAHIM

Here is a fantastic magical game with numbers 1 to 9.

Write the numbers 1 to 9 each digit only once in three rows such that the sum of the first two rows is equal to the third row. *All the digits 1 to 9 are used, each only once!*

How many such solutions can you find?
Perhaps you may need help of a computer!
To avoid duplicates, you might try placing the smaller digit in the first row, above the bigger digit in the second row. In the first example you might note that 1 < 3, 2 < 5 and 7 < 9.

Examples:

127	192	219
+359	+384	+438
486	576	657

*The author, an exponent of recreational mathematics, can be contacted at
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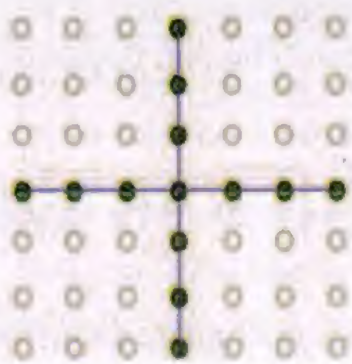
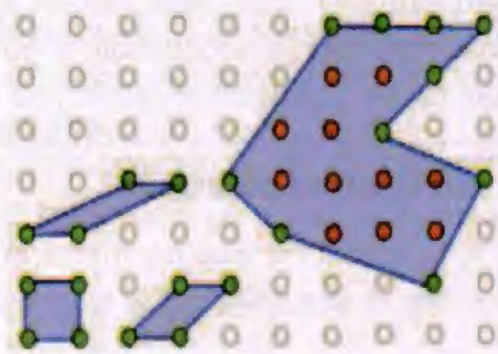
PICK'S THEOREM

(A report on a Project done by VIII standard students of Sarada School, Chennai)

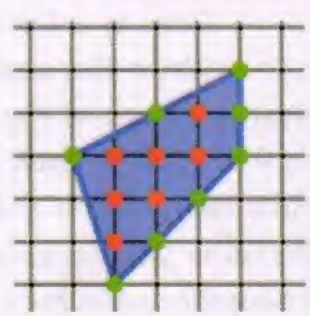
When we were asked to do a project on Pick's theorem, we found that Pick's theorem is not frequently used in Indian text books of mathematics for schools. The theorem, first proved by Georg Alexander Pick in 1899, is a classic result of geometry. To understand it*, first we need to figure out terms like a point lattice, a lattice polygon, a boundary point, an interior point, etc.





A point lattice is a regularly points spaced array of points. **Lattice points** are points whose coordinates are both integers, such as (2,5) , (-3,1) and (0,4).

A polygon whose vertices are of a point lattice is known a *lattice* polygon. This simply means the vertices have integer coordinates.



A boundary point is a lattice point on the polygon (including vertices). (Shown by green dots)

An interior point is a lattice point in the polygon's interior region. (Shown by red dots)

Pick's theorem offers an elegant formula to compute the area of a *simple* lattice polygon (whose boundary consists of a sequence of connected nonintersecting straight-line segments).

Ed.: *A neat investigative discussion is available on the web at http://jwilson.coe.uga.edu/emat6680fa05/schultz/6690/pick/pick_main.htm

There can also be other ways of finding the area of such a polygon, but Pick's theorem provides a tidy alternative.

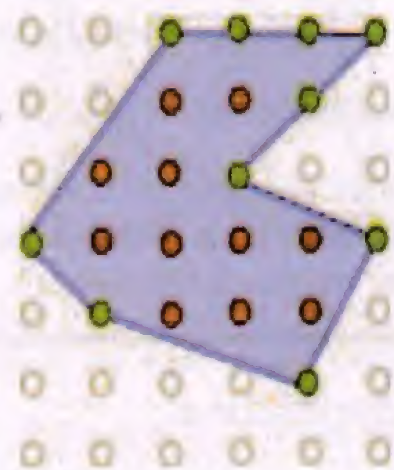
Pick's formula is given as

$$A = i + \frac{b}{2} - 1,$$

where, A is the area of the polygon, i = the number of interior points of the polygon and b = the number of its boundary points. A nice thing about the formula is that it is applicable to any polygon and not necessarily a convex polygon.

The above formula brings out a simple yet beautiful result that the area of a lattice polygon is always an integer or half an integer.

In the figure given,



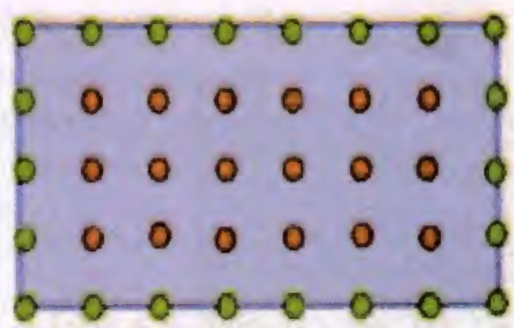
i = 11 (interior points)

b = 10 (boundary points)

Thus $A = i + \frac{b}{2} - 1 = 11 + \frac{10}{2} - 1 = 15.$

You can dissect the polygon into triangles (the process called triangulation) and verify this answer.

As another illustration, let us consider a rectangle of length l and height h.



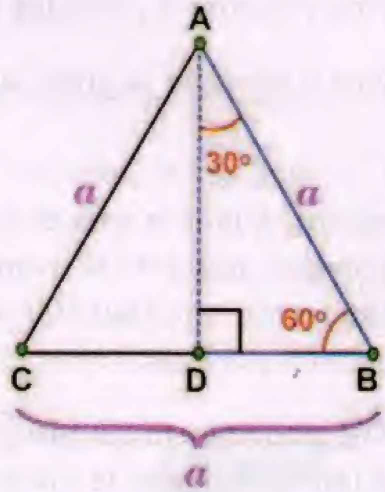
In this case,

$i = (l-1)(h-1); \quad b = 2l + 2h.$

$$\begin{aligned} A &= i + \frac{b}{2} - 1 \\ &= (l-1)(h-1) + \frac{2l+2h}{2} - 1 \\ &= lh - l - h + 1 + l + h - 1 \\ &= lh \end{aligned}$$

One can prove many interesting results using Pick's formula. Here is a sample:

It is impossible to draw an equilateral triangle with its vertices as lattice points.



The justification is simple: Assume that you can draw one such equilateral triangle; then its area must be an integer or a half-integer. If a is the length of a side of its side, its area has to be an integer A (which is $A=\frac{\sqrt{3}}{4}a^2$). Also, since AD and BD are both integers (the point D being the midpoint of BC), $a^2 = AD^2 + BD^2$ must be an integer. Thus in the expression for area $A=\frac{\sqrt{3}}{4}a^2$, A as well as a^2 are integers, leading to an absurd result that $\sqrt{3}$ must be an integer! This is a contradiction.

There seems to be many such wonderful results on the applications of Pick's formula. We feel that this theorem could be an exciting addition to our mathematics content.

Curiosities!

The prime number 73,939,133 has a very strange property. If you keep removing a digit from the right hand end of the number, each of the remaining numbers is also a prime. It is the largest number known with this property.

Take a look:
73939133
7393913
739391
73939
7393
739
73
7
are all prime!

Curiosities!

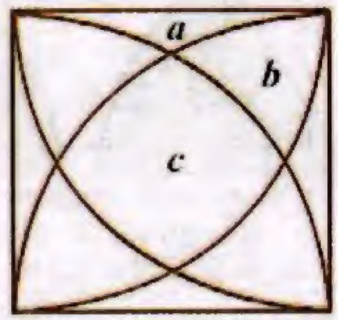
$$165{,}033 = 16^3 + 50^3 + 33^3.$$

$$1^n+6^n+8^n=2^n+4^n+9^n \text{ (for } n=1 \text{ or } 2).$$

TWO FASCINATING PROBLEMS

S. Sukumar, 36, South Car Street, Palani.

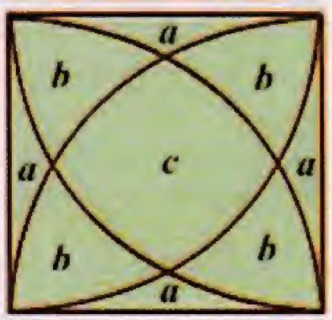
I wish to share with the readers of Junior Mathematician, two problems and their solutions which were quite absorbing according to me. These problems were a part of 'extra assignments' that were optional for us. Some students used higher concepts like trigonometry etc. to solve them; however the ones which are described here are found during my search on the web pages. The argument has been supplemented with diagrams and explanations.



Problem 1:

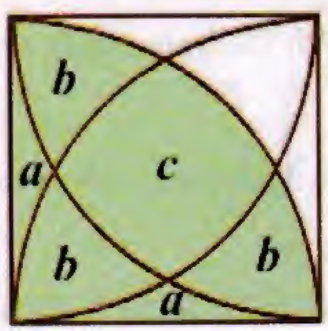
Consider a square. At each vertex of the square, draw an arc with radius equal to the side of the square. In the process, several curved regions are formed. Find the areas of regions ***a***, ***b*** *and* ***c*** *(shown in the figure).*

We try to construct three (simultaneous) equations in a, b and c. For easy handling, assume that the length of the side of the square is 2 units.



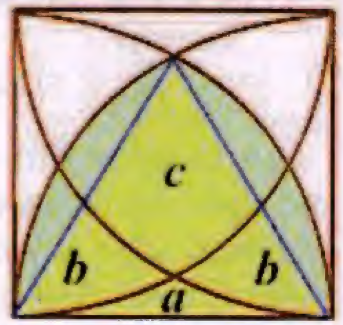
The entire square region = 4 times a, 4 times b and one time c. The total area of these four sub regions must be equal to the area of the square, which is 4 square units. Thus we get the first equation:

$4a + 4b + c = 4$ ....(1)



To form a second equation, we consider only the arc centred at the lower left corner and the region of the square wrapped by it. This sub region is one-fourth of a whole circle whose radius is 2 units. This must also be equal to two times a, 3 times b and one time c. We get the second equation as 2a+3b+c

=¼ the area of circle of radius 2 unit $= \frac{1}{4}\pi \times 2^2 = \pi$ ....(2)



Obtaining the third equation connecting a, b and c is somewhat challenging. Because of symmetry of arcs, the triangle shown is equilateral. Using the formula for an equilateral triangle, its area is $\frac{\sqrt{3}}{4}\times 2^2=\sqrt{3}$. The total area of the regions One time a + two times b and one time c is equal to twice the area of the sector (with one corner of square as centre, 60^o as its angle and 2 units radius) minus the area of the equilateral triangle.

Using the formula for each of the area of sectors $\frac{x}{360}\times\pi r^2 = \frac{60}{360}\times\pi\times 2^2=\frac{2}{3}\pi$ and we get

$$a + 2b + c = =2(\frac{2}{3}\pi)-\sqrt{3}=\frac{4}{3}\pi-\sqrt{3} \quad \text{...(3)}$$

The three equations to be solved are

$$4a + 4b + c = 4 \quad \text{..............................(1)}$$
$$2a + 3b + c = \pi \quad \text{..............................(2)}$$
$$a + 2b + c = \frac{4}{3}\pi-\sqrt{3} \quad \text{..................(3)}$$

2(2) — (1) gives $\quad 2b + c = 2\pi - 4$ ....(4)

2(3) — (2) gives $\quad b + c = 5\frac{\pi}{3}-2\sqrt{3}$ ....(5)

(4) — (5) gives $\quad b = \frac{\pi}{4}-4+2\sqrt{3}$

Put the value of b in (5) and solve for c. We get

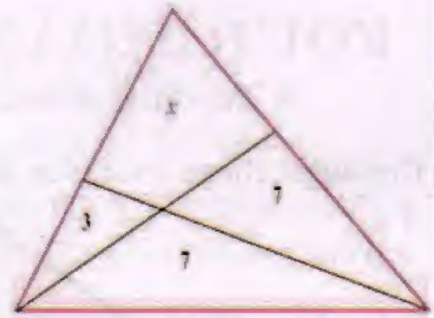
$$c = 4\frac{\pi}{3}+4-4\sqrt{3}$$

Substituting these values of b and c in any one of the three equations gives

$$a = -2\frac{\pi}{3}+4-\sqrt{3}$$

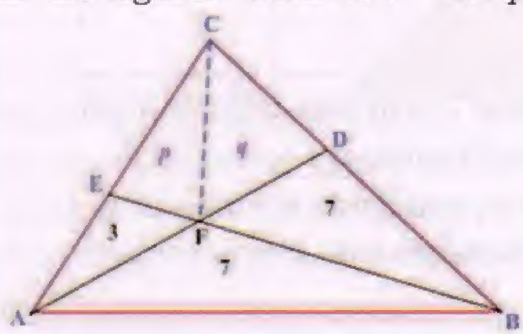
We thus solve the problem which appeared to be intimidating but in fact required only simple mathematics from school level.

Problem 2: (This problem also is deceptively simple and requires careful analysis).



Consider the given triangle. It is divided into four regions. The areas of three of the regions are marked. X is the area of the remaining region. Find X.

Name the vertices and the point of intersection in the interior, as shown in the figure. Join *CF*. Let p and q be the areas of ΔCEF and ΔCDF respectively. Thus the required X would be $p + q$.



Now area of ΔABF = area of ΔBDF. These triangles ΔABF and ΔBDF, have the same altitude and so their bases must be of same length; that is, $AF=DF$.

The altitudes of ΔACF and ΔCDF are of equal length and as seen in (1) we have now $AF = DF$. Therefore area of ΔACF = area of ΔCDF.

Thus we get $\qquad p + 3 = q$..................................(1)

Since ΔAEF and ΔABF have same altitude and their respective bases are EF and BF, we can state that $EF : BF = 3 : 7$.

Similarly, since ΔCEF and ΔCBF have same altitude and their respective bases are EF and BF, we can state that

The area of ΔCEF : the area of $\Delta CBF = 3 : 7$.

Therefore we can write $7p = 3(q+7)$ ....(2)

We now solve the equations (1) and (2).

Substituting for q in (2), we obtain

$$7p = 3(p+3+7).$$

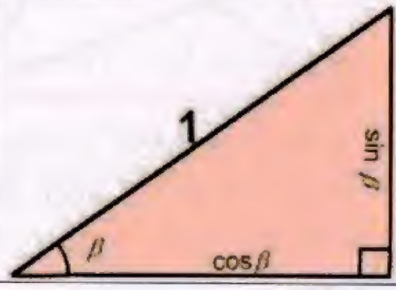
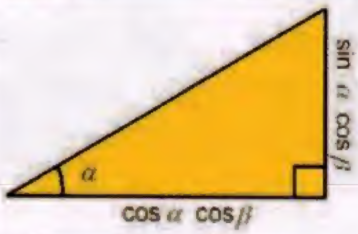
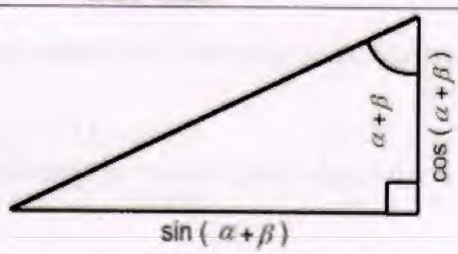
From this we find that $p = 7\frac{1}{2}$, which when substituted in (1) gives $q = 10\frac{1}{2}$.

Thus the required $X = p + q = 7\frac{1}{2} + 10\frac{1}{2} = 18$.

FOUR TRIANGLES AND A FORMULA

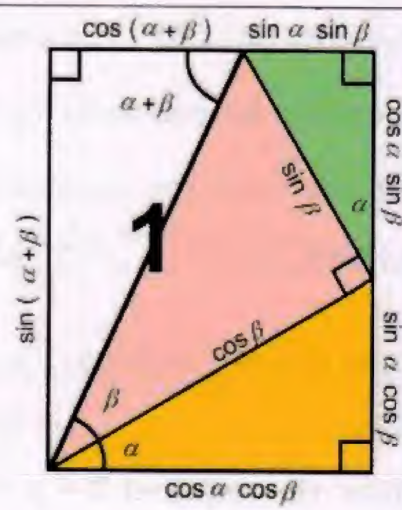
B. Naghabhushan, Tirumalainagar, Hastinapuram, Chennai.

Consider these triangles and their sides as indicated

1		Look at $\angle\beta$ in this right triangle. Take Hypotenuse to be = 1. $\therefore$ Opposite side is $\sin\beta$ Adjacent side is $\cos\beta$
2		Look at $\angle\alpha$ in this right triangle. Take Hypotenuse to be $\cos\beta$. $\therefore$ Opposite side is $\sin\alpha\cos\beta$ Adjacent side is $\cos\alpha\cos\beta$
3		Observe $\angle\alpha$ in this different triangle. Take Hypotenuse to be $\sin\beta$. $\therefore$ Opposite side is $\sin\alpha\sin\beta$ Adjacent side is $\cos\alpha\sin\beta$
4		Look at angle $(\alpha+\beta)$. Take Hypotenuse to be = 1. $\therefore$ Opposite side is $\sin(\alpha+\beta)$ Adjacent side is $\cos(\alpha+\beta)$

Place the four triangles in juxtaposition as shown. Now you see that

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos((\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$



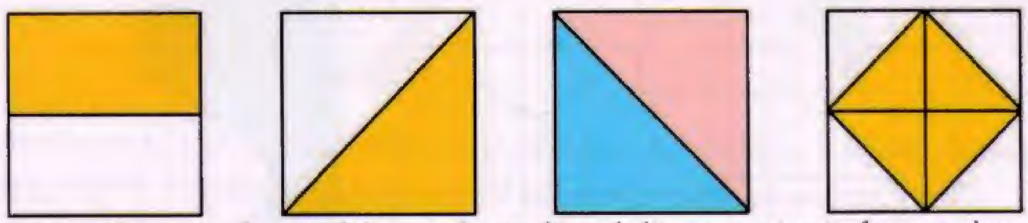
In diagrams showing multiple relations, it is always helpful to split it into several parts and visualize the same.

<u>An activity</u> :

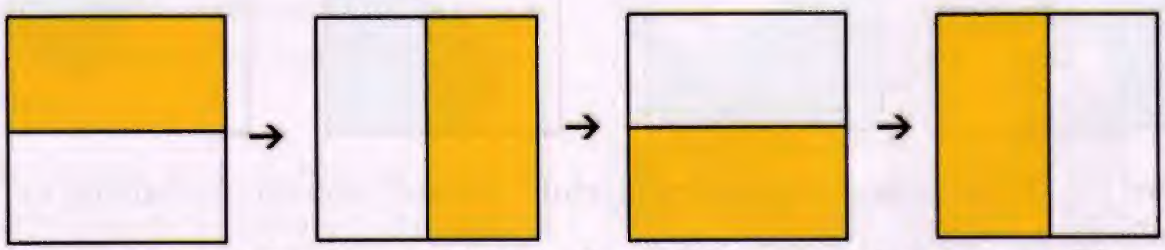
HALVING A SQUARE

R. Nandhini, Sarada Sec.School, Gopalapluram, Chennai

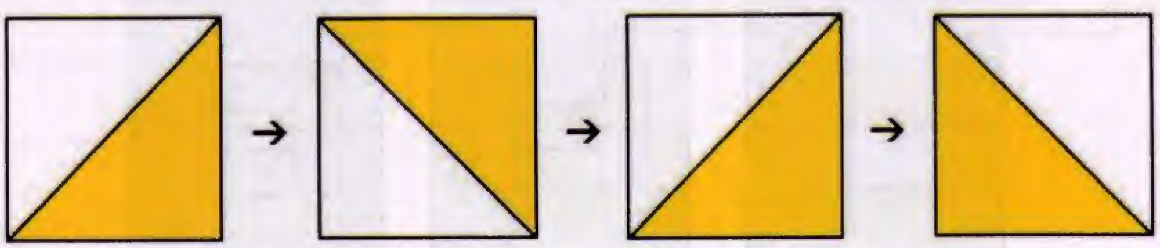
When a square region is given, in how many ways can you halve it? One immediately thinks of the 3 or 4 possibilities, like the ones shown here.



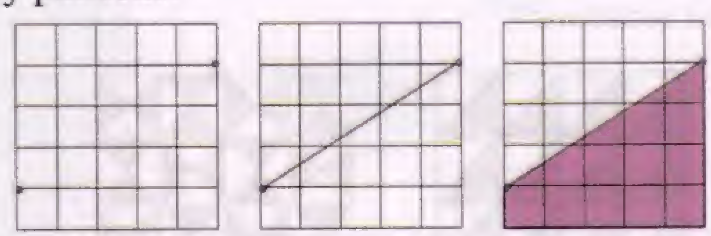
When our class took up this week-end activity, we started experimenting by drawing on dot sheets the above diagrams and found that our attempts had a lot to do with congruence and symmetry. We soon found that these can be easily renovated into equivalent reflections, rotations etc.



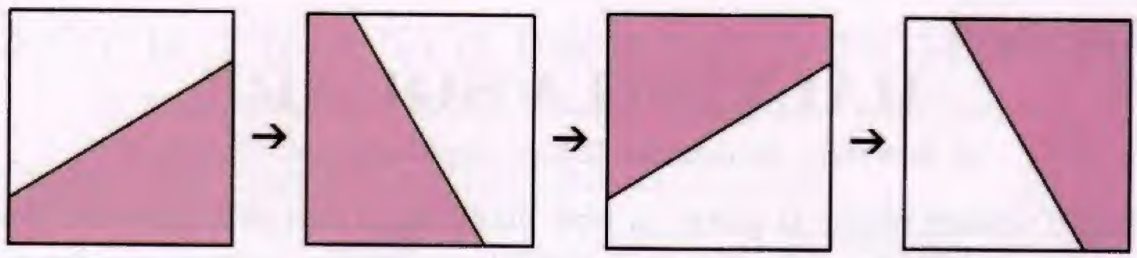
Some students (perhaps with help from their parents) came up with

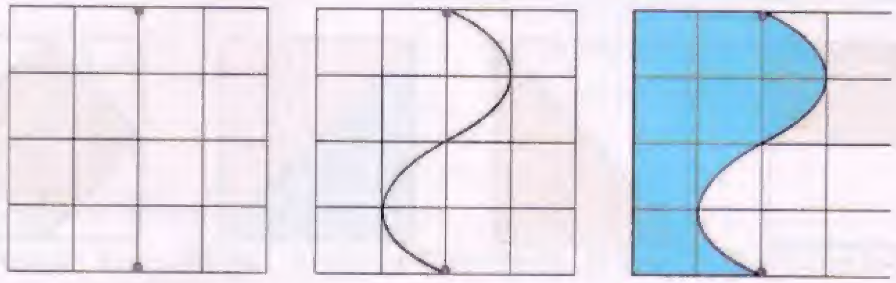


little more tricky patterns:

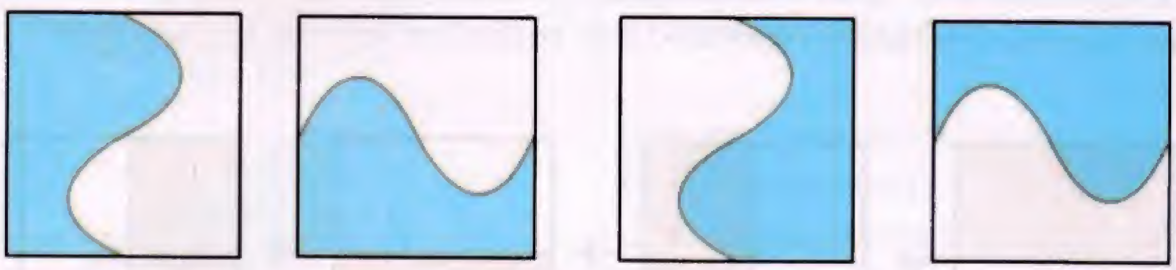


and could be transformed in several ways:

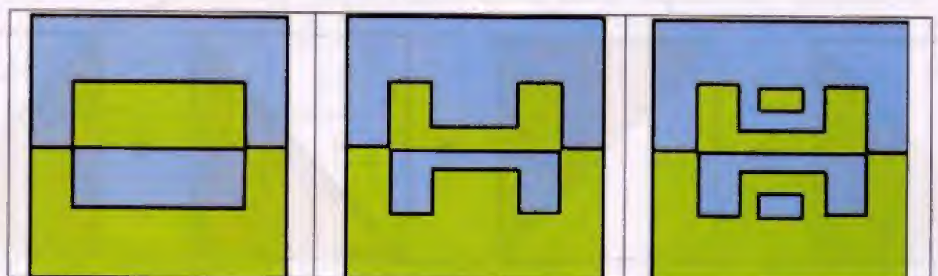




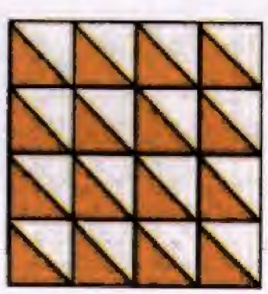
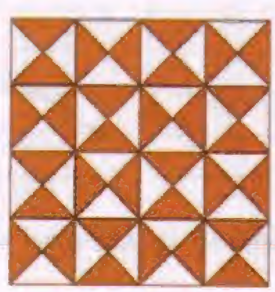
and these could also be transformed in several ways:



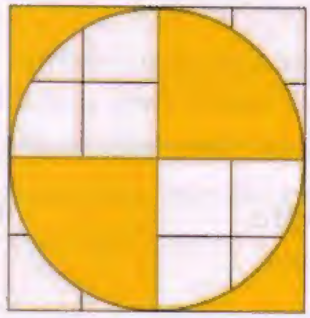
Are not the following 'halves' wonderful? We can also do the halving as follows:

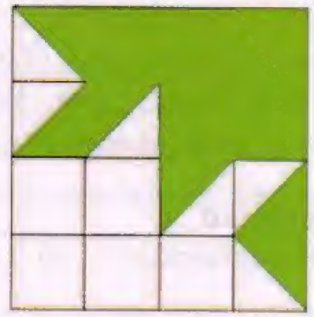


These are quite symmetric in design. Are you able to see the reflections, translations and rotations?

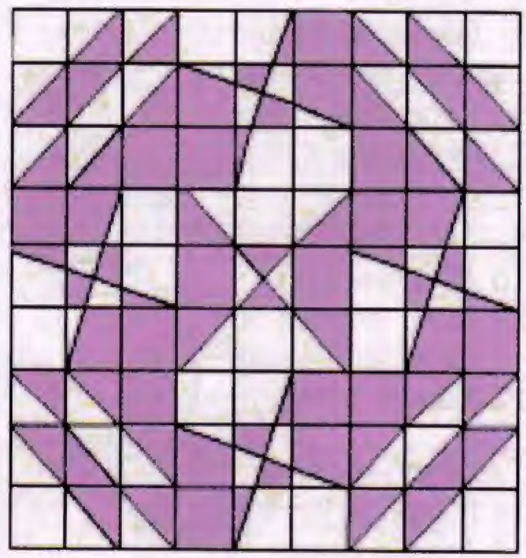
 

We did not even imagine these when we started this activity:





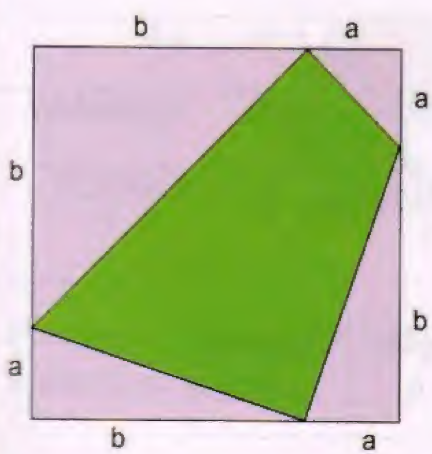
Can you guess how we verified if the coloured portions halved the square? Drawing them on dotted sheets and counting squares were really advantageous.

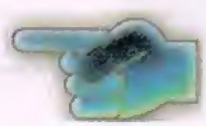


We found in our web search a very problematical diagram with lots of twists and turns:

Count the squares and check if the shaded portion is really half of the square!

Our teacher told us that we can extend this activity to a rectangle, a trapezium or even to a curved figure. Perhaps we try this during our vacation!





Ed:

How about this? Can you use some algebraic identity to show halving?

A MATCHING WORK OUT

This is not like the usual matching of information from two columns. You find three columns of data on this page. You have to match them suitably. The first one is solved as a sample. Answers can be found elsewhere in this journal.

Mathematician	Topic	Country
1 Dattaraya Ramchandra Kaprekar	*a)* Logarithms	i. USSR
2 Pythagoras	*b)* Number Theory	ii. FRANCE
3 David Hilbert	*c)* A Mathematician's Apology	iii. ENGLAND
4 Euclid	*d)* Coordinate Systems	iv. INDIA
5 Leonardo Bonacci — known as Fibonacci	*e)* Fourier transform	v. ENGLAND
6 Rene Descartes	*f)* Plane Geometry	vi. GREECE
7 Georg Cantor	*g)* Logic	vii. FRANCE
8 Bertrand Russell	*h)* Transfinite Numbers	viii. SCOTLAND
9 Karl Friedrich Gauss	*i)* $e^{i\pi} + 1 = 0$	ix. FRANCE
10 Al-Kowarizmi	*j)* Number Theory	x. INDIA
11 Isaac Newton & Gottfried Leibniz	*k)* Symbolic Logoic	xi. GERMANY
12 Nikolai Lobachevski	*l)* Abstract Algebra	xii. SWITZERL AND
13 Leonhard Euler	*m)* Complex Numbers.	xiii. USSR
14 Pierre de Fermat	*n)* Algebra	xiv. GERMANY
15 Amalie Emmy Noether	*o)* Non-Euclidean Geometry	xv. ENGLAND & GERMANY
16 George Boole	*p)* Calculus	xvi. ITALY
17 John Napier of Merchiston	*q)* Recreational Maths	xvii. ENGLAND
18 Godfrey Harold "G.H." Hardy	*r)* 23 Problems	xviii. US
19 Jean-Baptiste Joseph Fourier	*s)* Fibonacci series	xix. GERMANY
20 Subbayya Sivasankaranarayana Pillai	*t)* Metaphysics, Music & Mathematics	xx. GREECE

Sample solution:

Mathematician	Topic	Country
1	*q*	x
.	.	.
.	.	.

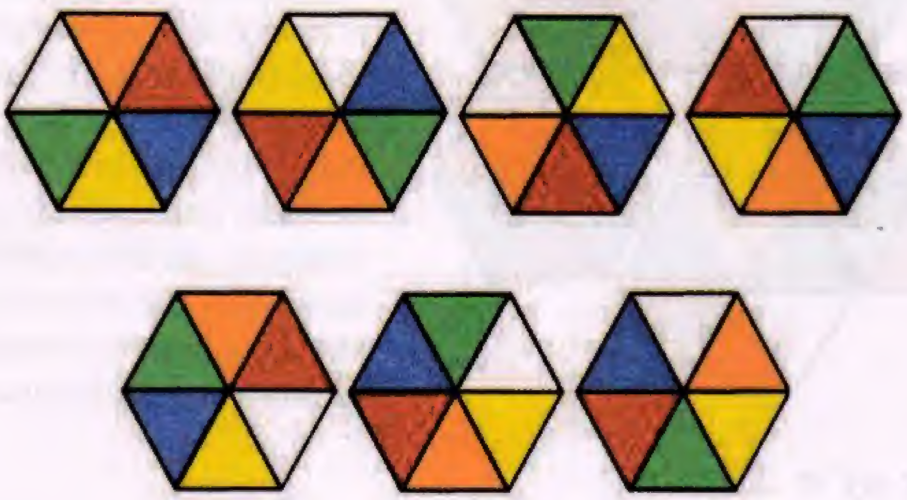
An activity :

HEXAGON TESSELLATION

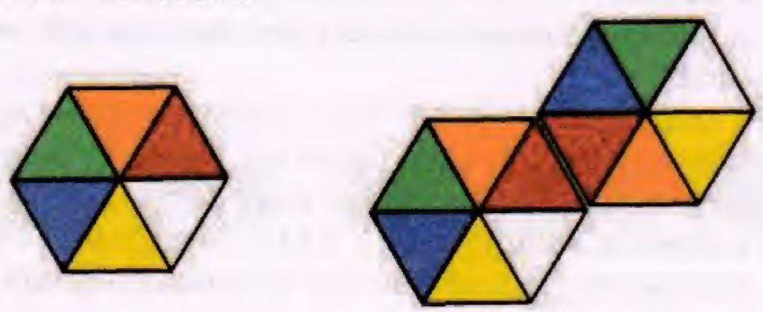
K. Bhaskar, Sourashtra School, Madurai

This special tessellation of making a regular hexagon with seven identical regular hexagons was one of the several items seen in an educational exhibition in our town.

You are given seven regular congruent hexagons. Each of them is divided into six identical equilateral triangles. The six triangles of *each* hexagon have different colours. Moreover, the colours are arranged in different orders.



The activity proceeds as follows:



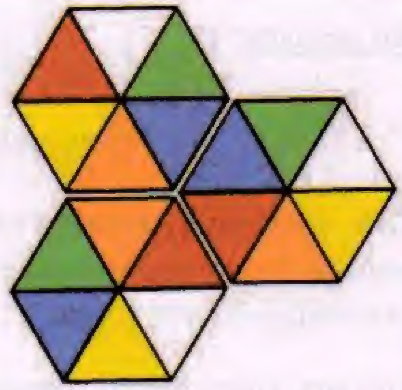
Step 1

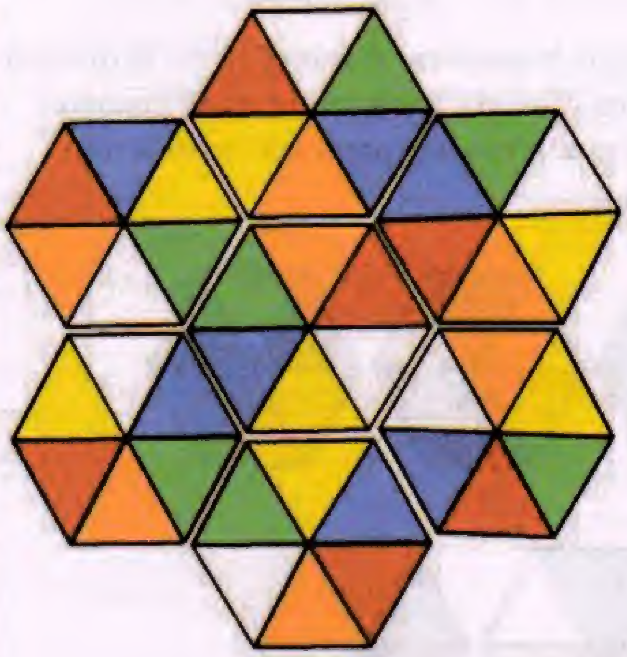
Fix one hexagon on a pane, say a plank or a table.

Step 2

Place a second hexagon such that at one of the edges adjacent sectors have same colour.

Continue with a third. Remember that every time the hexagons around the first fixed one must be such that the adjacent sectors have the same colour.





Here is a solution claimed by a student. Do you think that it satisfies all the conditions of the activity? Check yourself. Theoretically, the number of solutions is huge. Each hexagon can be put at each of the inner hexagon and each outer hexagon can be rotated in 6 ways. Now the calculation will show you how enormous is the number of possibilities!

Think of a Number...

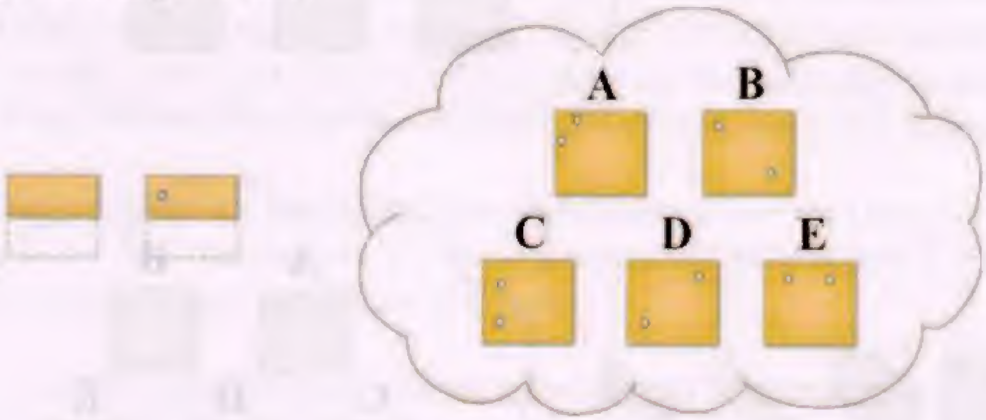
1. **Think of a number**. Add to it the next largest number. Add 5 to this sum. Divide by 2. Subtract the original number. Try this with different numbers. What do you find?
2. **Think of a number**. Triple it. Add the number that is one higher than the original number. Add 7 to this sum. Divide the answer by 4. Subtract 2. Try this with different numbers. What do you find?
3. **Think of a number**. Multiply it by 3. Add 8. Double this sum. Subtract 4. Divide by 6. Subtract the number you first thought of. Try this with different numbers. What do you find?
4. **Think of a number** greater than 3. Write down the numbers that are 2 smaller and 2 greater than the one you thought of. Find the product of these two numbers. Add 4. Find the square root of this sum. Try this with several numbers. What do you find?

An Activity:

FOLD-PUNCH-UNFOLD

M. Parvez, St.Xavier's convent, Gurgaon.

We had a funny activity during our arts and crafts class. Later the teacher explained how it is related to mathematics, surprising us! It all involves folding a square paper, perforating a hole using a Single Punch Machine <u>*only once*</u>, opening it and then verifying a guess regarding the activity. A sample is here:

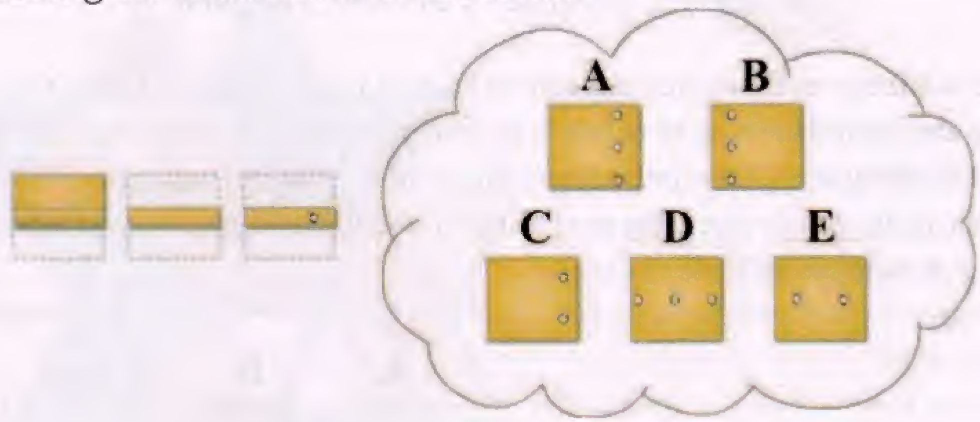


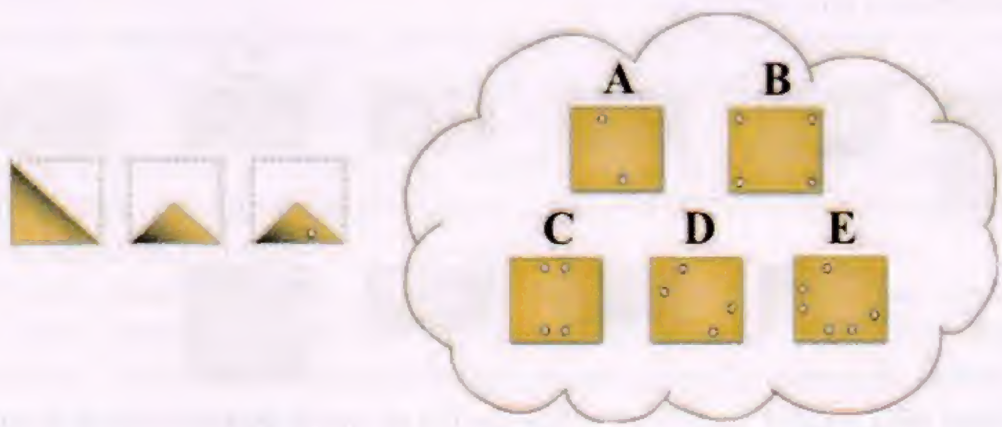
On the left you find how a square paper is folded and punched; in the cloud several options are given. Only one of them is correct. You have to choose the correct option and then actually verify if your option is correct by performing the activity.

The class was asked to try with two or three such guesses and activities.

Some of them were quite easy while some were really challenging. In the end we even tried to reverse the process. One of us was asked to produce a punched version and others were asked to explain how one should fold the paper to get such a punch. After instruction by the teacher, we even tried to identify the line of symmetry.

Here are some of the problems we had; on the option to be identified is given among the alternatives in the cloud.





BEGINNING WITH THE ENDING

Kollutla Maniram, Suryapet, Telangana

Sometimes we begin with certain assumption or plan and start working on how to take it to some interesting or useful outcome. But there are occasions when we concentrate on the end-product we want and try to plan suitably. Suppose you have to catch a train at 9 pm on a given day, you concentrate on things to be done before reaching station; perhaps you may tell yourself: "The taxi will take half an hour to reach station, before that I will need 15 minutes for dinner and my packing the luggage will take 20 minutes preceding dinner..." and so on. Finally you will decide that you would have to get ready, say, by 6 pm. Mathematics also reminds us about this peculiarity. Given a sumin mathematics, we look at the conditions given and try to use them to arrive at certain result; there are also instances when we focus more on the solution we want and try to work backwards! This we understood during our class today.

The problem we discussed was as follows: *Given that the sum of two numbers is 4 and the product of t he very same two numbers is 5, find the sum of the reciprocals of the two numbers.*

It was an algebra class and so we naturally started assuming that the two numbers are a and b; thus we had two equations.

$a + b = 4$ ....(1) and $ab = 5$...................(2)

From (1), we find $b = 4 - a$ and hence (2) could be written as

$$a(4-a)=5$$

or $$a^2-4a+5=0.$$

Using formula, we get $a=\dfrac{4\pm\sqrt{16-20}}{2}$

$$=\frac{4\pm\sqrt{-4}}{2}=\frac{4\pm\sqrt{(-1)(4)}}{2}=\frac{4\pm 2i}{2} \text{ where } i \text{ stands for } \sqrt{-1}$$

$$=2\pm i$$

Let us take $a = 2+i$ and $b = 2 - i$.

$\therefore$ Sum of the reciprocals of a and b

$$= \frac{1}{a}+\frac{1}{b}=\frac{1}{2+i}+\frac{1}{2-i}=\frac{(2-i)+(2+i)}{(2+i)(2-i)}=\frac{4}{2^2-i^2}=\frac{4}{4+1}=\frac{4}{5}.$$

When we felt that we have successfully solved this problem, our teacher quietly went to the blackboard and wrote

Sum of the reciprocals of a and $b = \frac{1}{a}+\frac{1}{b}=\frac{b+a}{ab}=\frac{4}{5}.$

While we had labored for quite some time to get the answer, she got it in a minute! We were tempted by assuming the numbers and manipulating them while she deliberated on the end-product and immediately saw that working backwards would yield a compact method.

We now understand how reverse method is very helpful in getting elegant solutions. Here is one more problem we solved:

Abdul's father is 48 years old. He is 15 years older than 3 times Abdul's age. How old is Abdul?

Routine method	*Shorter "backward working" method*
Let Abdul's age $= x$ (years.) 3 times Abdul's age $= 3x$ 15 years more than this $= 3x+15$ This must be equal to 48. $\therefore$ we get the equation $3x+15=48$ $3x+15=48$ means $3x+15-15=48-15$ $3x=33$ Divide both sides by 3. $\frac{3x}{3}=33$ Therefore, $x=11$ Age of Abdul $=11$ years.	To get from Abdul's age to his fathers age, we multiply Abdul's age by 3 and add 15. While working backwards, we start with the father's age, subtract 15 and divide by 3. $\therefore Abdul's\ age=\frac{father's\ age-15}{3}$ $=\frac{48-15}{3}$ $=\frac{33}{3}=11.$

We find that working from the end-product in the reverse direction is also a very useful line of attack in many cases. What is 'done' by the problem-proposer is 'undone' by this procedure.

A suggestive Activity:

PLAY SHADOWS, MATHEMATICALLY!

(An account of need to study shadows is given here by Sri V. Sundaramurthy, AMTI)

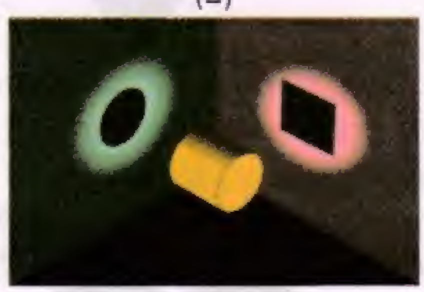
Shadows excite us very much. For little children this is always true. When they look at shadows, they look for patterns, size and then relative positions. Is it possible to make a longer shadow or a shorter shadow than that is observed? Is the shadow always the same shape as the object? Can you make a shadow fool your friend so he cannot guess what the original object is? These are few things that surface. These make a lot of mathematics. And in later years, the concept of projection further strengthens these notions.

To start with, use flashlight and study the different possibilities that any shape makes. You may take a moment to trace the shadows too.

(1)

(2)

(3)



(4)

In (4) the shadows are cast by light source in two different directions. Are you able to identify them?

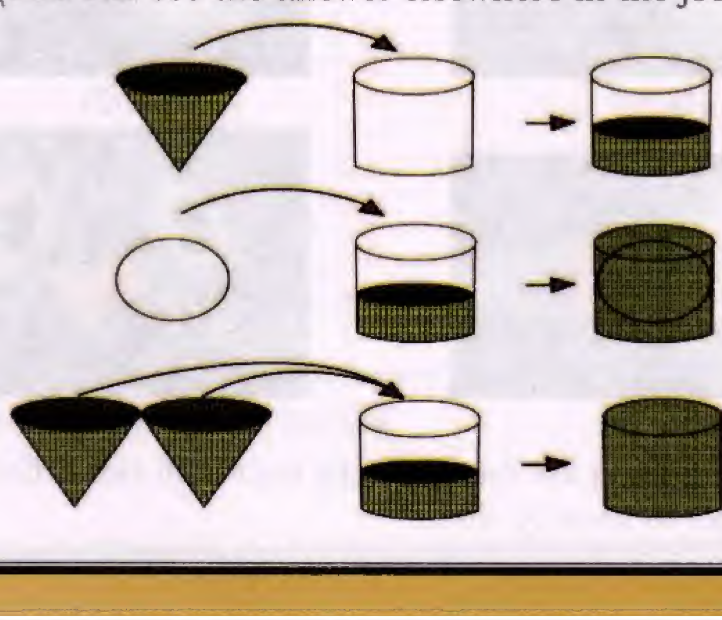
Shadows can appear in countless shapes and variations. Here are shadows of some 3D geometrical shapes. What could they be? What could they be?

Several questions can be discussed. Can you get a rectangular shadow from a cube? A hexagonal shadow? A triangular shadow? A trapezoidal shadow? Atriangular prism can cast a square shadow; how? Try this activity and you will certainly come across many more wonderful ideas.

Why should we study shadows? It is the basic concept behind the notion of Projection in mathematics. The idea of projection has its roots in the phenomenon of shadows cast by real world objects on the ground.

VISUALZING RELATIONSHIPS

Examine the sequence of drawing given here and guess the relationships; you would have studied mathematical formulae concerning these, but here is an explanation through diagrams. The cylinder, cone and the sphere all have the same radii and same height. The cylinder and the cone have an open base; the shading corresponds to water being poured or displaced. (You can see the answer elsewhere in the journal.)



A PROBLEM FROM "*LILAVATI*"

(Taken from "The Mathematics Teacher (India)" Vol.V No.3 & 4) published by the Association of Mathematics Teachers of India. The following is a portion of a beautiful article writen by Late veteran Prof T. Todadri Iyengar)

Problems in *Lilavati* (Bhāskara II's treatise on mathematics, written in 1150) appear poetic in content and form. Here is one sample.

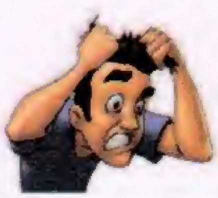
Partha Karna vadhaya marghana ganam
krudhdho ranae sandhathae
Thasyardhena nivarya thatchcharagnam
moolaischathurbirhayan
Salyam shadbiratheshubisthribirapi
cathram dhwajam karmukhom
Chichchedhasya sira sarena kathithae
yan Arjuna sondhathae.

Partha (*Arjuna*) angrily let fly a quiver of arrows to slay *Karna*. With half the arrows, he countered *Karna*'s. With four times the square-root of the number in the quiver, he killed the horses; with six arrows, he slew *Salya* (charioteer of *Karna*); with three arrows he destroyed the umbrella, the flag and the bow; with one (the last) he cut off *Karna*'s head. How many were the arrows in the quiver?

One usual method of solution would be to take y^2 to be the n umber required. We get the quadratic equation

$$\frac{y^2}{2}+4y+6+3+1=y^2 \text{ which reduces to } y^2-8y-20=0.$$

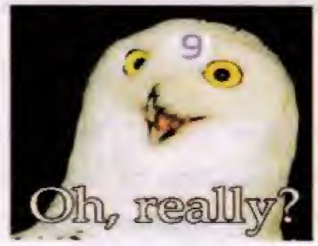
As y has to be positive, y = 10 and the number in the quiver is 100.



What is wrong?

"Among all the natural numbers, 1 is the biggest". You know, why? Let N be any natural number other than 1. Then $N^2>N$. So N cannot be the biggest natural number, since there is a still bigger natural number, namely N^2. This argument will apply to all numbers except 1. So no number other than 1 can be the biggest natural number. (Clarification: Elsewhere in this issue)

NINE (9) IS FINE!



The largest single-digit number is 9. It has many fascinating characteristics. Here we see some of them.
In the first place is the attraction of pattern of 9 times table.

Consider the nine times table.

Multiplication	Digit-sum of Product
$9 \times 1 = 9$	9
$9 \times 2 = 18$	$1 + 8 = 9$
$9 \times 3 = 27$	$2 + 7 = 9$
$9 \times 4 = 36$	$3 + 6 = 9$
$9 \times 5 = 45$	$4 + 5 = 9$
$9 \times 6 = 54$	$5 + 4 = 9$
$9 \times 7 = 63$	$6 + 3 = 9$
$9 \times 8 = 72$	$7 + 2 = 9$
$9 \times 9 = 81$	$8 + 1 = 9$
$9 \times 10 = 90$	$9 + 0 = 9$

- Here is something to do with angle measures:

Sum of digits

Complete angle = 360^0 — $3 + 6 + 0 = 9$

Straight angle = 180^0 — $1 + 8 + 0 = 9$

A right angle = 90^0 — $9 + 0 = 9$

Half of a right angle = 45^0 — $4 + 5 = 9$

Half of $45^0 = 22.5^0$ — $2 + 2 + 5 = 9$

Similarly we can continue like

Half of $22.5^0 = 11.25$ $(1 + 1 + 2 + 5 = 9)$,

Half of $11.25^0 = 5.625^0$ $(5 + 6 + 2 + 5) = 18 \Rightarrow 1 + 8 = 9)$

etc.

Solution for A MATCHING WORK OUT

1	2	3	4	5	6	7	8	9	10
q	*t*	*r*	*f*	*s*	*d*	*h*	*g*	*m*	*n*
x	*vi*	*xviii*	*xx*	*xvi*	*ii*	*xiv*	*xvii*	*xix*	*i*

11	12	13	14	15	16	17	18	19	20
p	*o*	*i*	*j*	*l*	*k*	*a*	*c*	*e*	*b*
xv	*xiii*	*xii*	*vii*	*xi*	*v*	*viii*	*iii*	*ix*	*iv*

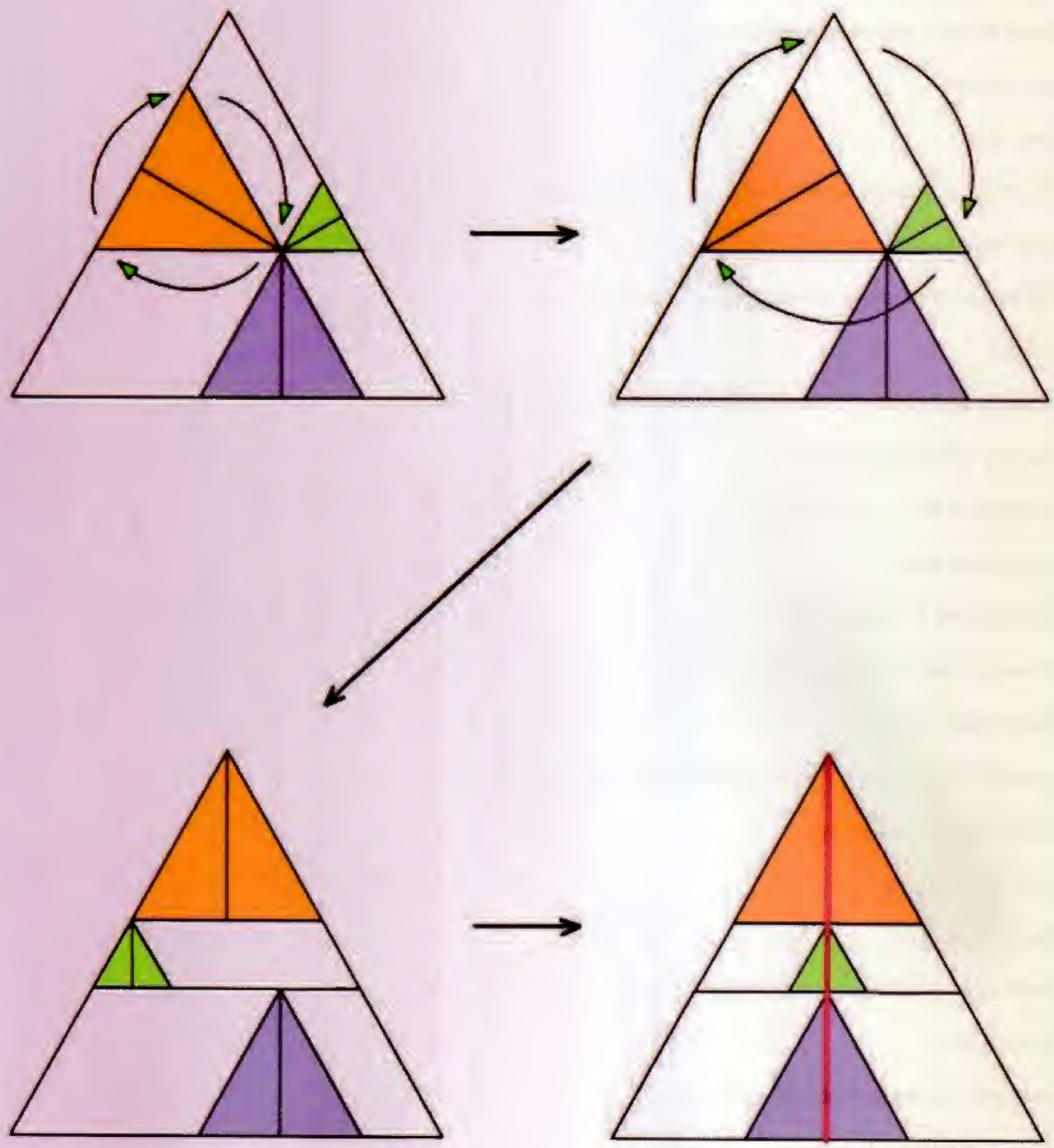
Solution for VISUALZING RELATIONSHIPS

Comparing volumes, $V_{cone} : V_{sphere} : V_{cylinder} = 1 : 2 : 3$

Solution for **What is wrong?**

The mistake lies in assuming that there is a biggest natural number.

A Picture Story



VIVIANI'S THEOREM

The sum of distances of a point inside an equilateral triangle or on one of its sides equals the length of its altitude.

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